Curriculum Vitae of Professor Patrizia Daniele

General Info

- Date and place of birth: April 14th, 1973, Augusta (SR) - Italy
- Address: Department of Mathematics and Computer Sciences
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Education

- Degree in Mathematics, University of Catania, July 15th, 1995. Thesis: "Variational Inequalities and Duality"
- PhD in Applied Mathematics and Computer Sciences, University of Naples "Federico II", March 20th, 2000. Thesis: "Variational Inequalities and Duality. Applications to Equilibrium Problems"

Positions

- **Visiting Scholar** for the spring semester from March 1th to May 31th, 2006 at the Division of Engineering and Apllied Science (DEAS), Harvard University.
- **Associate Professor** in Operations Research, University of Catania, from January 18th, 2005.
- **Researcher** MAT/09 (Operations Research), University of Catania, from October 1st, 2001.
- Winner of a national competition for an "**Assegno di Ricerca**" on "Calculus of the Variations and Applications to Equilibrium Problems", Faculty of Mathematical, Physical, and Natural Sciences, University of Catania, November 2nd, 1999 – September 30th, 2001.
- **Professional Adviser** for the study of Lagrange Theory, Duality Theory, Minimax Theory related to more general dual formulations of the traffic equilibrium problem, University of Pisa, July-November 1995.

National and International Awards

- Award "Gioacchino Iapichino" for the year 1996, Accademia Nazionale dei Lincei.
- Rockefeller Foundation Bellagio Center Fellowship, 2003.

- Visiting Scholarship during the Spring semester 2006 at the Division of Engineering and Applied Science (DEAS), Harvard University.

Organizing Activity

- Member of the Organizing Committee of the International Workshop **"Equilibrium Problems and Variational Models"**, Taormina, December 3-5, 1998.
- Member of the Organizing Committee of the International School of Mathematics "G. Stampacchia" 32nd Workshop **"Equilibrium Problems and Variational Models"**, Erice, June 23 – July 2, 2000.
- Member of the Organizing Committee of the First Joint Meeting AMS-UMI Special Sessions: **"Nonlinear Elliptic and Parabolic Equations and Systems"** and **"Variational Analysis and Applications"**, Pisa, June 12-16, 2002.
- Member of the Organizing Committee of the International Conference on **"Variational Analysis and Applications"**, Erice, June 20-30, 2003.
- Organizer of the Minisymposium **"Equilibrium Problems and Variational Models"** during the 5th International Congress on Industrial and Applied Mathematics, Sydney, July 7-11, 2003.
- Member of the Organizing Committee of the Workshop on **"Equazioni a Derivate Parziali: Aspetti Metodologici, Modellistica, Applicazioni"**, Ragusa Ibla, June 29 - July 2, 2005.
- Member of the Organizing Committee of the International School of Mathematics "G. Stampacchia" 44th Workshop on **"Variational Analysis and Partial Differential Equations"**, Erice, July 5-14, 2006.
- Co-organizer of the Workshop **"Complex Networks – Equilibrium and Vulnerability Analysis with Applications"**, Catania, March 10-12, 2008, together with Professor Anna Nagurney.
- Member of the Organizing Committee of the International School of Mathematics "G. Stampacchia" 51st Workshop on **"Variational Analysis and Applications"**, Erice, May 9-17, 2009.
- Organizer of the session **Variational Inequalities and Applications to Economic Market Models**, XXIV EURO Conference, Lisbon, July 11-14, 2010.
- Member of the Organizing Committee of the **Fifth China-Italy Colloquium on Applied Mathematics CICAM5,** Acireale (CT), September 27-30, 2010.
- Co-organizer of the session **"Variational Inequality Problems and Equilibrium Problems"**, 41st Annual Conference AIRO, Villa San Giovanni, September 7-10, 2010, together with Dr. Laura Scrimali.

Other Scientific Activities

- Host of Professor Anna Nagurney, John F. Smith Memorial Professor of the University of Massachusetts at Amherst, during the Fulbright Senior Specialist Award in Business Administration at the University of Catania, March 5-19, 2008.

- Member of the Editorial Board of the Series "New Dimensions in Networks", Edward Elgar Publishers Ltd., Editor: Prof. Anna Nagurney, University of Massachusetts.

- Member of the working group EQUILIBRIUM MODELS in COMPLEX SYSTEMS (EMICS), coordinator Prof. Igor V. Konnov (http://emics.ksu.ru/).

- Member of the Teaching Committee of the PhD in Mathematics for Technology, University of Catania.

- Referee for journals (Journal of Optimization Theory and Applications, Operations Research Letters, European Journal of Operational Research, Quantitative Finance, Nonlinear Analysis, Computational Management Science, Mathematics and Computers in Simulation, Journal of Systems Science and Systems Engineering, Le Matematiche, Indian Journal of Pure and Applied Mathematics, Optimization Letters).

- Visiting position at the University of Massachusetts, Amherst, Department of Finance and Operations Management, Eugene M. Isenberg, School of Management, guest of Prof. Anna Nagurney, January 2-5 2001 and July 9-10, 2004.

Research Projects

- Ex 60% 2003 Project – Existence problems, uniqueness and regularity for elliptic and parabolic equations and systems, coordinator Prof. M. Marino.
- FIRB Project – Analysis of linear and nonlinear partial differential equations: methods, models, applications, coordinator Prof. M. Marino.
- PRIN 2008 - Variational Analysis and Partial Differential Equations, coordinator Prof. A. Maugeri.

Teaching

- Operations Research, University of Catania, A.Y. 2001-2002, A.Y. 2002-2003, A.Y. 2003-2004, A.Y. 2004-2005, A.Y. 2005-2006, A.Y. 2007-2008, A.Y. 2008-2009, A.Y. 2009-2010, A.Y. 2010-2011.
- Optimization, University of Catania, A.Y. 2005-2006, A.Y. 2006-2007, A.Y. 2007-2008, A.Y. 2008-2009, A.Y. 2009-2010.
- Network and Supernetwork, University of Catania, A.Y. 2005-2006, A.Y. 2006-2007, A.Y. 2007-2008, A.Y. 2008-2009, A.Y. 2009-2010.
- Mathematical Analysis, University of Catania, A.Y. 2001-2002, A.Y. 2002-2003, A.Y. 2003-2004, A.Y. 2004-2005.
- Themes of Operations Research for the Scuola Interuniversitaria Siciliana di Specializzazione per l'Insegnamento nella Scuola Secondaria, University of Catania, A.Y. 2002-2003, A.Y. 2003-2004, A.Y. 2004-2005.

- Themes of Numerical Analysis for the Scuola Interuniversitaria Siciliana di Specializzazione per l'Insegnamento nella Scuola Secondaria, University of Catania, A.Y. 2001-2002.

Research Activity

- Traffic Equilibrium Problem and its Dual;

- Traffic Equilibrium Problem in the Continuum case;
- Price equilibrium problem for spatially distributed markets;
- Equilibrium problems on time-dependent networks;
- Supply-Demand markets equilibrium problem;
- Duality theory and non regular Lagrange theory applied to the previous problems;
- Gap functions and equivalent optimization problems;
- Numerical computation based on the direct method and a generalized subgradient method for Variational Inequalities;
- Dynamics of Complex Networks in an Environment of Risk and Uncertainty;
- Financial equilibrium problems;
- Relationship between Evolutionary Variational Inequalities and Projected Dynamical Systems;
- Electric power networks
- Infinite-dimensional duality
- Discretization procedure and numerical computation for evolutionary variational inequalities
- Time-dependent supernetworks and variational formulations

Invited Talks and Seminars

- Workshop on Equilibrium Problems with Side Constraints. Lagrange Theory and Duality, Scilla (RC) May 17-18, 1996;
- Conference on Numerical Methods for Optimization, Cortona (AR) June 8-12, 1997;
- 9th International Conference on Differential Equations, Plovdiv (Bulgaria) August 18-23, 1998;
- Workshop on Equilibrium Problems and Variational Models, Taormina (ME) December 3-5, 1998;
- Conference on Evolution Equations and Applications, Cortona (AR) May 9-14, 1999;
- Conference Giornate Nonlineari, Bologna November 11-13, 1999;
- 32nd Workshop on Equilibrium Problems and Variational Models, Erice (TP) June 23 – July 2, 2000;
- 2001 SIAM Annual Meeting, San Diego (California), July 9-13, 2001;
- XXXII Annual Confernce AIRO "La Ricerca Operativa nella Gestione del Territorio e delle Risorse", Villasimius (CA), September 4-7, 2001;
- VI National Congress of the Società Italiana di Matematica Applicata e Industriale, Chia Laguna (CA) May 27-31, 2002, Session: Applications of Variational Inequalities (Coordinator: Prof. I.V. Konnov); First Joint Meeting AMS-UMI, Pisa June 12-16, 2002;
- International Conference on "Variational Analysis and Applications", Erice June 20-30, 2003;

- 5th International Congress on Industrial and Applied Mathematics, Sydney July 7-11, 2003;
- Conference on High Performance Algorithms and Software for Nonlinear Optimization - Status and Perspectives, Ischia, June 18-20, 2004;
- Fourth World Congress of Nonlinear Analysts WCNA-2004, Orlando, Florida, USA, June 30 – July 7, 2004;
- XXXV Annual Conference AIRO, Lecce September 7-10, 2004; 8th SIAM Conference on Optimization, Stoccolma (Svezia), May 15-19, 2005;
- 5th ISAAC Congress, Catania July 25-30, 2005;
- XXXVI Annual Conference AIRO, Camerino September 6-9, 2005;
- "Southwestern Ontario Dynamics Day Workshop", Guelph, Ontario, Canada April 7, 2006;
- Applied Math Colloquium Series at Harvard University, Boston, Massachusetts, USA, May 25, 2006;
- International Conference "Variational Analysis and Partial Differential Equations", Erice July 5-14, 2006;
- 6th International Congress on Industrial and Applied Mathematics (ICIAM 2007), Zurich July 16-20, 2007;
- Workshop on "Complex Networks – Equilibrium and Vulnerability Analysis", Catania March 10-12, 2008;
- XXXIX Annual Conference AIRO, Ischia September 7-11, 2008;
- Joint Mathematics Meetings, Washington DC January 5-8, 2009;
- 51th Workshop on "Variational Analysis and Applications", Erice May 9-17, 2009;
- 23rd European Conference on Operational Research, Bonn July 5-8, 2009;
- Fifth China-Italy Colloquium on Applied Mathematics CICAM5, Acireale (CT) September 27-30, 2010.

Publications

Books

[1] Editor of the book "Equilibrium Problems and Variational Models", Kluwer Academic Publishers, in collaboration with F. Giannessi – A. Maugeri, 2002.

[2] Author of the book "Dynamic Networks and Evolutionary Variational Inequalities", Edward Elgar Publishing, 2006.

Book Chapters

[3] P. Daniele, A. Maugeri, "Vector Variational Inequalities and Modelling of a Continuum Traffic Equilibrium Problem", Vector Variational Inequalities and Vector Equilibria, F. Giannessi Ed., Vol. 38, 2000, pp. 97-111.

[4] P. Daniele, “Variational Inequalities for Static Equilibrium Market. Lagrangean Function and Duality”, Equilibrium Problems: Nonsmooth Optimization and Variational Inequality Models, Kluwer Academic Publishers, F. Giannessi – A. Maugeri – P. Pardalos Eds., 2001, pp. 43-58.

[5] P. Daniele, A. Maugeri, “On Dynamical Equilibrium Problems and Variational Inequalities”, Equilibrium Problems: Nonsmooth Optimization and Variational Inequality Models, Kluwer Academic Publishers, F. Giannessi – A. Maugeri – P. Pardalos Eds., 2001, pp. 59-69.

[6] P. Daniele, A. Maugeri, “Equilibrium Problems and Variational Inequalities: a Continuum Transportation Model”, Computational fluid and solid mechanics, Vol. 1, 2 (Cambridge, MA, 2001), Elsevier, Amsterdam, 2001, pp. 1543-1545.

[7] P. Daniele, G. Idone, A. Maugeri, “The Continuum Model of Transportation Problem”, Equilibrium Problems and Variational Models, Kluwer Academic Publishers, P. Daniele – F. Giannessi – A. Maugeri Eds., 2002, pp. 53-60.

[8] P. Daniele, A. Maugeri, “The economic Model for Demand-Supply Problems”, Equilibrium Problems and Variational Models, Kluwer Academic Publishers, P. Daniele – F. Giannessi – A. Maugeri Eds., 2002, pp. 61-78.

[9] P. Daniele, “Variational Inequalities for Evolutionary Financial Equilibrium”, Innovations in Financial and Economic Networks, A. Nagurney Ed., 2003, pp. 84-108.

[10] P. Daniele, “Variational Inequalities for General Evolutionary Financial Equilibrium”, in Variational Analysis and Applications, F. Giannessi - A. Maugeri Eds., Springer Verlag, 2005, pp. 279-299.

[11] M.G. Cojocaru, P. Daniele, A. Nagurney, “Projected Dynamical Systems, Evolutionary Variational Inequalities, Applications, and a Computational Procedure”, in Pareto Optimality, Game Theory and Equilibria, A. Chinchuluun - A. Migdalas - P.M. Pardalos - L. Pitsoulis Eds., Nonconvex Optimization and its Applications Series (NOIA), Springer, Berlin, Germany, 2008, pp. 803-836.

[12] P. Daniele, S. Giuffré, A. Maugeri, S. Pia, “A panoramic View on Projected Dynamical Systems”, in Nonlinear Analysis and Variational Problems, P. Pardalos - Th.M. Rassias - A.A. Khan Eds., Springer, 2009, 235–258.

Papers

[13] P. Daniele, “Dual Variational Inequality and Applications to asymmetric traffic equilibrium problem with capacity constraints”, “Equilibrium Problems with Side Constraints. Lagrangean Theory and Duality”, F. Giannessi - A. Maugeri editors, Le Matematiche, 49, 1994, pp. 211-222.

[14] P. Daniele, “Duality Theory for Variational Inequalities”, Communications in Applied Analysis 1, no. 2, 1997, pp. 257-267.

[15] P. Daniele, “A remark on a dynamic model of a Quasi-Variational Inequality”, Rendiconti del Circolo Matematico di Palermo, Serie II, Suppl. 48, 1997, pp. 91-100.

[16] P. Daniele, A. Maugeri e W. Oettli “Variational Inequalities and Time-dependent traffic equilibria”, C. R. Acad. Sci, Paris, t. 326, serie 1, 1998, pp.1059-1062.

[17] P. Daniele, A. Maugeri, W. Oettli, “Time-dependent traffic equilibria”, Jou. Optim. Th. Appl., Vol. 103, No. 3, 1999, pp. 543-555.

[18] P. Daniele, “Lagrangean Function for Dynamic Variational Inequalities”, Rendiconti del Circolo Matematico di Palermo, Serie II, Suppl. 58, 1999, pp. 101-119.

[19] P. Daniele, A. Maugeri, “Variational Inequalities and Discrete Continuum Models of Network Equilibrium Problems”, Mathematical and Computer Modelling **35,** 2002, pp. 689-708.

[20] P. Daniele, “Evolutionary Variational Inequalities and Economic Models for Demand Supply Markets”, M^3AS: Mathematical Models and Methods in Applied Sciences, **4 (13)**, 2003, pp. 471-489.

[21] P. Daniele, “Time-Dependent Spatial Price Equilibrium Problem: Existence and Stability Results for the Quantity Formulation Model”, Journal of Global Optimization, **28**, 2004, pp. 283-295.

[22] P. Daniele, G. Idone, A. Maugeri, “Variational Inequalities and the Continuum Model of Transportation Problem”, International Journal of Nonlinear Sciences and Numerical Simulation 4, 2003, pp. 11-16.

[23] M.G. Cojocaru, P. Daniele, A. Nagurney, “Projected Dynamical Systems and Evolutionary Variational Inequalities via Hilbert Spaces and Applications”, Jou. Optim. Th. Appl., 27, no. 3, 2005, pp. 1-15.

[24] M.G. Cojocaru, P. Daniele, A. Nagurney, “Double-layered dynamics: a unified theory of projected dynamical systems and evolutionary variational inequalities”, European Journal of Operational Research, 175, 1, 2006, pp. 494-507.

[25] P. Daniele, S. Giuffrè, S. Pia, “Competitive Financial Equilibrium Problems with Policy Interventions”, Journal of Industrial and Management Optimization, Vol. 1, no.1, 2005, pp. 39-52.

[26] P. Daniele, “Evolutionary Variational Inequalities Applied to Financial Equilibrium Problems in an Environment of Risk and Uncertainty”, Nonlinear Analysis, **63**, 2005, pp. 1645-1653.

[27] A. Nagurney, Z. Liu, M.G. Cojocaru, P. Daniele, “Dynamic Electric Power Supply Chains and Transportation Networks: An Evolutionary Variational Inequality Formulation”, Transportation Research E., **43**, 2007, pp. 624-646.

[28] P. Daniele, S. Giuffrè, “General Infinite Dimensional Duality Theory and Applications to Evolutionary Network Equilibrium problems”, Optimization Letters, **1**(3), 2007, pp. 227-243.

[29] A. Nagurney, D. Parkes, P. Daniele, "The Internet, Evolutionary Variational Inequalities, and the Time-Dependent Braess Paradox**"**, Computational Management Science, **4**, 2007, pp. 243-281.

[30] P. Daniele, S. Giuffrè, G. Idone, A. Maugeri, "Infinite Dimensional Duality and Applications", Mathematische Annalen, **339** (1), 2007, pp. 221-239.

[31] P. Daniele, "Lagrange Multipliers and Infinite-Dimensional Equilibrium Problems", Journal of Global Optimization, **40** (1), 2008, pp. 65-70.

[32] P. Daniele, S. Giuffrè, A. Maugeri, **"**Remarks on General Infinite Dimensional Duality with Cone and Equality Constraints", Communications in Applied Analysis, **13** (4), 2009, pp. 567-578.

[33] P. Daniele, "Evolutionary Variational Inequalities and Applications to Complex Dynamic Multi-level Models", Transport. Res. Part E, 46, 2010, 855-880, doi: 10.1016/j.tre.2010.03.005.

[34] A. Barbagallo, P. Daniele, A. Maugeri, "Variational Formulation for a General Dynamic Financial Problem: Balance Law and Liability Formula", Nonlinear Analysis: Theory, Methods & Applications, doi: 10.1016/j.na.2010.10.013.

Submitted

[35] P. Daniele, S. Giuffrè, A. Maugeri, F. Raciti, "Duality Theory and Applications to Unilateral Problems".

Proceedings

[36] P. Daniele, "Variational Inequalities, Equivalent Optimization Problems and Associated Lagrangean Function", 9^{th} Int. Coll. on Differential Equations, D. Bainov (Ed.) VSP, Utrecht, The Netherlands, 1999, pp.107-112.

PhD Thesis

[37] P. Daniele, "Disequazioni Variazionali e Dualità. Applicazioni a problemi di equilibrio", Tesi di Dottorato di Ricerca in Matematica Applicata ed Informatica – XI Ciclo - Università degli Studi di Napoli "Federico II".

Publications results for "Author=(daniele)"

MR2273417 (2008e:49002)

Daniele, Patrizia(I-CATN-MI)

Dynamic networks and evolutionary variational inequalities.

New Dimensions in Networks. *Edward Elgar Publishing Limited, Cheltenham,* 2006. xii+253 pp. ISBN: 978-1-84376-929-3; 1-84376-929-8

49-02 (49J40 90B10 91B50)

Networks play a major role in theoretical issues of a variety of research areas and in many practical aspects of modern life. This book deals with dynamic networks, i.e., networks such that (i) the structure is time-invariant but (ii) the phenomena (economical, physical, etc.) occurring in them vary with time. This is the case, e.g., with traffic models and their travel demands and with financial models and their investments in assets and liabilities. The second ingredient of the book is evolutionary variational inequalities.

The book's scope consists in developing the strict connection between equilibrium problems in dynamic networks and evolutionary variational inequalities. This is accomplished by a recipe that focuses on the so-called user-optimization approach, in which each user aims at minimizing his personal cost: he tries to obtain an equilibrium that nobody is interested in changing. The point is the following: time-dependent equilibrium conditions can be expressed in terms of evolutionary variational inequalities and the latter provide solution methodologies. Time-dependent variational inequalities were introduced by J.-L. Lions and G. Stampacchia [Comm. Pure Appl. Math. **20** (1967), 493--519; MR0216344 (35 \#7178)]. The theory of such inequalities for traffic equilibrium networks was first developed by P. Daniele, A. Maugeri and W. Oettli [C. R. Acad. Sci. Paris Sér. I Math. **326** (1998), no. 9, 1059--1062; MR1647182 (99g:90041); J. Optim. Theory Appl. **103** (1999), no. 3, 543--555; MR1727254 (2000g:90008); A. Maugeri, Matematiche (Catania) **49** (1994), no. 2, 305--312 (1995); MR1397308 (97i:90103)], and F. Raciti [in *Equilibrium problems and variational models (Erice, 2000)*, 369--377, Kluwer Acad. Publ., Norwell, MA, 2003; MR2043482 (2004k:90012); Appl. Math. Lett. **17** (2004), no. 2, 153--158; MR2034761 (2004j:49024); J. Optim. Theory Appl. **122** (2004), no. 2, 387--403; MR2093849 (2005e:91064)].

The "ambient space" for the flow trajectories is the Lebesgue space $L^p([0,T],\Bbb R^q)$; the equilibrium solution is shown to be a trajectory belonging to such a space. This brings into play an infinite-

dimensional context and requires tools from functional analysis.

Computational procedures are provided to compute solutions. To this end, various methods are considered: the direct method, the subgradient method, a discretization method, and a technique originating from projected dynamical systems theory. Finally, the sensitivity of solutions is investigated.

The book is organized into five chapters and six appendices.

Chapter 1 is an introduction to the contents of the book and the approach adopted therein. Chapters 2 through 5 have a common structure: they start with the variational formulation, then give existence theorems and discuss computational procedures, finally providing examples. Chapter 2 deals with traffic network models (the static case, the dynamic case, and models with additional constraints and retarded data). Chapter 3 focuses on the evolutionary spatial price equilibrium. Both the price formulation approach and the quantity formulation approach are used; a sensitivity analysis is made, also. Chapter 4 considers the evolutionary financial equilibrium problem (first a model with a quadratic utility function is adopted, then a model with a more general utility is considered; finally, a model including policy intervention is investigated). Chapter 5 is a self-contained treatment of projected dynamical systems theory; it emphasizes the connections between such a theory and the theory of evolutionary variational inequalities.

Appendix A gives definitions and basic properties for functions, multifunctions, and cones. Appendix B contains material on weak convergence. Appendix C is devoted to derivatives: directional derivatives, Gateaux and Fréchet derivatives, and subdifferentials. Appendix D describes basic concepts and gives definitions about variational inequalities, first in finite dimension and then in infinite dimension. Quasi-variational inequalities are recalled in Appendix E, separately for the finite-dimensional and the infinite-dimensional case. Finally, Appendix F is devoted to infinite-dimensional duality.

As mentioned in the Acknowledgments, a part of Section 4.1 appeared in [P. Daniele, in *Variational analysis and applications*, 279--299, Springer, New York, 2005; MR2159978 (2006d:49011)] and a part of Section 4.2 appeared in [P. Daniele, in *Innovations in financial and economic networks*, 84--109, Edward Elgar, Cheltenham, 2003].

Contents of Chapters 2--5:

Chapter 2 considers the basic traffic equilibrium problems. Transportation networks were first studied in 1920 by A. C. Pigou [*The economics of welfare*, Macmillan, London, 1920] but only later were network equilibrium problems considered. The seminal work, due to J. G. Wardrop ["Some theoretical aspects of road traffic

research", Proc. Inst. Civil Engineers, Part II **1952**, 325--378; per bibl.], appeared in 1952; in 1956 the approach presented therein was given a rigorous mathematical foundation by M. J. Beckmann, C. B. McGuire and C. B. Winsten [*Studies in the economics of transportation*, Yale Univ. Press, New Haven, CT, 1956]. In 1979, M. J. Smith [Transportation Res. Part B **13** (1979), no. 4, 295--304; MR0551841 (81a:90089)] showed that the equilibrium solution can be expressed via variational inequalities. The latter work represents the departure point of investigations of the traffic equilibrium problem. Delay effects and elastic models were introduced by T. L. Friesz et al. [Oper. Res. **41** (1993), no. 1, 179--191; MR1205096 (93j:90038)], Raciti [in *Equilibrium problems: nonsmooth optimization and variational inequality methods*, 247--253, Kluwer Acad. Publ., Dordrecht, 2001; MR2026136 (2004j:90026)] and L. Scrimali [Math. Models Methods Appl. Sci. **14** (2004), no. 10, 1541--1560; MR2095302 (2005h:90022)].

Theorem 2.2.1 characterizes the equilibrium by means of an evolutionary variational inequality. Corollary 2.3.1 gives three sufficient conditions for the existence of a solution. To make the model more realistic, traffic controls (modelling, e.g., traffic lights and one-way paths) are considered in Section 2.4. Theorem 2.4.1 gives a characterization of the equilibrium problem in this more general setting. As to the calculation of the solution, two approaches are considered in Section 2.5: the subgradient method and a discretization procedure.

Section 2.6 considers delay effects in traffic models. The motivation is the following: the network users require some time to adjust their paths, since the information travels in the network at a finite speed. Correspondingly, one has a "retarded equilibrium flow", which in Theorem 2.6.1 is characterized by means of a variational formulation. Sufficient conditions for the existence of a solution are given in Theorem 2.6.3.

Chapter 3 includes the presence of supplies and deals with two approaches to this problem: the price formulation (Section 3.1) and the quantity formulation (Section 3.2).

The price formulation is first considered in the static case, to obtain some hints useful in facing the dynamic case. The model considers a single commodity produced at n supply markets (each with a supply price) and consumed at m demand markets (each with a demand price). For each supply market there is a total supply and for each demand market there is a total demand. A transportation cost is associated with a unit of commodity transported between each supply-market pair. Upper bounds on the transportation flows are included. The model is called the "disequilibrium model", to emphasize the presence of supply and demand excesses.

Theorem 3.1.1 characterizes the equilibrium of the static model in terms of variational inequalities. Theorem 3.1.3 gives a characterization of the solution to the variational inequality as a saddle point of a suitably-defined Lagrangian. For the dynamic case, a characterization is provided in Theorem 3.1.4; three different conditions are given, guaranteeing the existence of a solution to the variational inequality associated with the spatially-distributed market problem. Computational procedures (direct method and discretization method) are considered for an example in Section 3.1.3.

The quantity formulation is used in Section 3.2, where it is shown that the time-dependent equilibrium conditions can be incorporated directly into an evolutionary variational inequality, for which an existence theorem is proved. Theorem 3.2.1 provides the characterization of the time-dependent market equilibrium in terms of a variational inequality. Three different sufficient conditions for the existence of a solution are given in Theorem 3.2.2. A sensitivity analysis of the model is stated in Theorem 3.2.3.

The last part of the chapter (Section 3.3) is devoted to the economic model for demand-supply markets.

Chapter 4 proposes a model that allows for time-dependent variance-covariance matrices associated with risk perception, financial volumes held by the sectors, optimal portfolio compositions, and instrument prices. The model takes the hint from previous work by A. Nagurney, J. Dong and M. Hughes [Optimization **26** (1992), no. 3-4, 339--354; MR1236617 (94h:90009)] but, instead of modeling the dynamics by using the projected dynamical systems theory, as done therein, it exploits evolutionary variational inequalities, which have an infinite-dimensional nature.

For a quadratic utility function, Section 4.1 states the model and gives equilibrium conditions, the variational inequality formulation, theoretical results, and a computational procedure. In particular, Theorems 4.1.2 and 4.1.5 are a characterization and an existence result, respectively. Section 4.2 deals with the same issues for a more general utility function. Theorems 4.2.2 and 4.2.5 are the corresponding characterization and existence results. Section 4.3 incorporates, into the model of Section 4.2, policy interventions, i.e., taxes and price controls. Theorem 4.3.4 is a characterization result. Section 4.4 illustrates a numerical example of a financial nature, which presents the important feature that the variance-covariance matrices, the lower and upper bounds on the prices of the financial instruments, and the financial volumes held by a sector can be all time-varying.

Chapter 5 is devoted to the theory of Projected Dynamical Systems (PDS), introduced in the finite-dimensional context by P. G. Dupuis and Nagurney [Ann. Oper. Res. **44** (1993), no. 1-4, 9--42;

MR1246835 (94k:49009)] and briefly reviewed in Section 5.1.

Section 5.2 turns to the infinite-dimensional framework and proves the equivalence between the solutions to an evolutionary variational inequality and the critical points of a projected dynamical system; the main result is Theorem 5.2.4. Section 5.3 gives a common formulation for the traffic network problem considered in Chapter 4, the quantity formulation of the spatial price equilibrium of the first part of Chapter 3, and the financial equilibrium problem of Chapter 4.

The theory of PDS is developed in Section 5.4; Proposition 5.4.2 is a uniqueness result that requires less restrictive assumptions than those previously existing in the literature. Section 5.5 formalizes via Theorem 5.5.1 the relationship between the "large-scale dynamics" of the evolutionary variational inequalities and the "small-scale dynamics" of the PDS. A computational procedure is addressed in Section 5.6.

Finally, Section 5.7 presents some numerical examples from transportation science, solved through the direct method and/or the discretization procedure.

Reviewed by Marcello Sanguineti

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